Lec 13; 10/09/2013

Big-Bang Nucleosynthesis:

As we saw, at $T < 1$ MeV ($t > 1$ sec) the weak interactions freeze.

One consequence of this freeze out is that reactions that convert p to n will be inefficient. As a result, the neutron to proton ratio keeps decreasing because of neutron decay:

$$\left(\frac{n}{p}\right)_{(t)} = \left(\frac{n}{p}\right)_0 \exp\left(-\frac{t - 1\,\text{sec}}{\tau_n}\right) \qquad t \gtrsim 1\,\text{sec}$$

$\tau_n \to$ neutron lifetime $\simeq 881$ sec

In addition, neutrons and protons can also fuse to form Deuteron:

$$n + p \rightleftarrows D + \gamma$$

The binding energy of D is ~ 2.2 MeV. Therefore photons with an energy $\gtrsim 2.2$ MeV can disintegrate D back to n and p. As we will see later, the ratio of baryons to photons in the universe follows:

$$\eta \equiv \frac{n_B}{n_\gamma} \sim 6 \times 10^{-10}$$

This implies that for every baryon there exist $\sim 10^9$ photons in the primordial plasma. These photons have a black-body spectrum. As long a for each D there is at least one photon with an energy $E_\gamma \gtrsim 2.2$ MeV, the photodissociation occurs efficiently. Using the Maxwell-Boltzmann distribution (as a good approximation), the number of photons with an energy above E is given by:

$$n_\gamma(E) \sim \exp\left(-\frac{E}{T}\right)$$

The temperature at which there is one photon with energy $E_\gamma \gtrsim 2.2$ MeV per baryon is thus found to be:

$$T \sim \frac{2.2 \text{ MeV}}{\ln\left(\frac{n_\gamma}{n_B}\right)} = \frac{2.2}{\ln(\eta^{-1})} \text{ MeV} \quad (\mathrm{I})$$

There is no D formation above this temperature as photodissociation is quite efficient. This is called the D bottleneck. After using the numerical value of η, we find $T_{NUC} \sim 100$ keV. One therefore finds the following time sequence after weak interactions freeze:

(1) $100 \text{ keV} < T < 1$ MeV: Weak interactions have already frozen.
($1 \text{ sec} < t < 100$ sec)

Neutrons decay to proton resulting in a decreasing value of

$\frac{n}{p}$:

$$\left(\frac{n}{p}\right)_{(t)} = \left(\frac{n}{p}\right)_0 \exp\left(-\frac{t - 1\,\text{sec}}{\tau_n}\right)$$

Nuclear reactions do not lead to formation of elements heavier than the Hydrogen because the first product, which is D, is immediately disintegrated by photons in the primordial plasma.

(2) $T < 100$ keV (100 sec $< t < 180$ sec): Deutron formation becomes possible:

$$n + p \rightarrow D + \gamma$$

Upon production, D participates in other reactions that lead to ^4He production:

$$D + D \rightarrow {}^4He + \gamma$$

$$D + D \rightarrow {}^3He + n \qquad {}^3He + D \rightarrow {}^4He + p$$

$$D + p \rightarrow {}^3He + \gamma$$

$$D + D \rightarrow {}^3H + p \qquad {}^3H + D \rightarrow {}^4He + n$$

We note that fusion of two nuclei occurs via quantum tunneling, and the corresponding rate is given by:

$$\langle \sigma v_{rel} \rangle \propto \exp\left[-2\bar{A}^{\frac{1}{3}}(Z_1 Z_2)^{\frac{2}{3}}\left(\frac{T}{1\,MeV}\right)^{-\frac{1}{3}}\right]$$

Here A_1, A_2 denote the atomic number of two nuclei $\left(\bar{A} \equiv \frac{A_1 A_2}{A_1 + A_2}\right)$ and Z_1, Z_2 are the number of protons in the two nuclei. All the above reactions are very efficient until $t \sim 180$ sec, which results in almost all neutrons ending up in ^4He nuclei. The mass fraction of ^4He is determined by $\frac{n}{p}$ at the end of the D bottleneck:

$$\frac{n}{p} = \left(\frac{n}{p}\right)_0 \exp\left(-\frac{100-1}{\tau_n}\right) \sim \frac{1}{7}$$

We note that:

$$n_{^4He} = \frac{1}{2}n \quad , \quad n_H = p - 2n_{^4He}$$

The ratio of mass in ^4He to the total mass (defined as Y_p) is therefore given by:

$$Y_p = \frac{4\, n_{^4He}}{4\, n_{^4He} + n_H} = \frac{2n}{p+n} = \frac{2\left(\frac{n}{p}\right)}{1+\left(\frac{n}{p}\right)} \simeq 0.250 \qquad (\mathrm{II})$$

This is a very important predictions of the big-bang model, which was made about 70 years ago. As we will see later, it is in very good agreement with observations.

A tiny number of D and ^3He remain as free nuclei. The reason being that the rate for nuclear reactions decreases as the universe expansion lowers T and the number density of participating nuclei. Once the number of free ^3He and D becomes very small, the reactions that fuse them to ^4He ($D + D \to {}^4He + \gamma$ and $^3He + D \to {}^4He + p$) become inefficient.

The predicted values of D and ^3He (relative to that of H) is given by:

$$\frac{D}{H} \sim O(10^{-5}) \qquad \frac{^3He}{H} \sim O(10^{-5} - 10^{-4}) \qquad (\mathrm{III})$$

In addition to D, ^3He, ^4He there is a very small amount of

^7Li made. The relevant reactions are:

$^3He + {}^4He \rightarrow {}^7Be + \gamma \qquad {}^7Be + e^- \rightarrow {}^7Li + \nu_e \qquad {}^7Be + n \rightarrow {}^7Li + p$

$^3H + {}^4He \rightarrow {}^7Li + \gamma$

The resulting abundance of ^7Li is predicted to be:

$$\frac{^7Li}{H} \sim O(10^{-10} - 10^{-9}) \qquad (\text{IV})$$

^7Li is the last stable element that is synthesized during the first 3 minutes in an appreciable amount. Heavier nuclei are not formed because of the drop in the nuclear reaction rates (due to expansion) and increasing Coulomb barriers in these reactions.

The predicted values of the primordial abundance of light elements depend on one free parameter η. Inferring these abundances from observations will allow us to find an experimentally consistent range for η. We will discuss this in detail soon.

The most sensitive dependence on η occurs for D and ^3He. We note that a smaller value of η (more photons per baryon) results in a lower T at the end of D bottleneck. Formation and fusion of D happens later in this case that, as argued before, results in larger number of free D and ^3He nuclei. Even a small change in η can lead to a significant change in D and ^3He abundances. It turns out that:

$$\frac{D}{H}, \frac{^3He}{H} \sim \eta^{-n} \quad (n \sim 1-2) \qquad (\nabla)$$

On the other hand, the ^4He abundance mildly depends on η. Actually, this dependence appears through the effect of η on the value of $\frac{n}{p}$ at the end of D bottleneck. Larger η (less photons per baryon) results in an increase in T at the end of D bottleneck, which is equivalent to this occuring at an earlier time. This leads to a larger value of $\frac{n}{p}$, and hence an increase in Y_p. We note that T at the end of D bottleneck has a logarithmic dependence on η, see Eq. (I). Therefore Y_p changes logarithmically with η. However, Y_p has a more sensitive dependence on the number of relativistic degrees of freedom g_* at $T \sim 1$ MeV. One can show that:

$$Y_p = 0.230 + 0.025 \log\left(\frac{\eta}{10^{-10}}\right) + 0.0075\,(g_* - 10.75) + 0.014\,(\tau_{n,\frac{1}{2}} - 10.6 \text{ min}) \qquad (\text{VI})$$

Here $g_* = 2 + \frac{7}{8} \times (4 + 2N_\nu^{eff})$ and $\tau_{n,\frac{1}{2}}$ is the halflife of the neutron. It is seen that $g_* > 10.75$ (equivalently $N_\nu^{eff} > 3$) results in an increase in Y_p. This can be understood as a larger g_* implies freeze out of weak interactions at a higher temperature (than ~ 1 MeV) thus a larger value of $\frac{n}{p}$. The dependence of Y_p on g_* in Eq. (VI) can be recast in the following form: $\Delta Y_p \sim 0.013\, \Delta N_\nu^{eff}$, where $\Delta N_\nu^{eff} = N_\nu^{eff} - 3$.